RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College under University of Calcutta)

B.A./B.SC. THIRD SEMESTER EXAMINATION, DECEMBER 2013

SECOND YEAR

Computer Science (Honours)

Paper : III

Date : 14/12/2013

Time : 11 am – 3 pm

Full Marks : 75

(Use separate answer book for each group)

Group – A

Answer **any one** question out of question no. 1 - 2. (1 × 10)

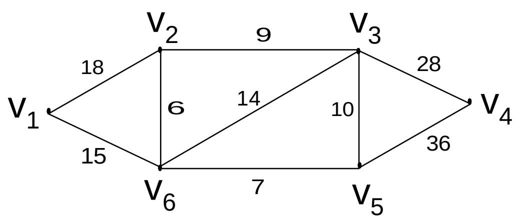
1. a) Define walk, path and circuit with suitable example. (3)

   b) Prove that a simple graph with n vertices and k components cannot have more than $\frac{(n-k)(n-k+1)}{2}$ edges. (4)

   c) How many extra bridges would be necessary to build in koinsberg bridge so that an Euler cycle would exist? Illustrate. (3)

2. a) If a connected planar graph G has n vertices, e edges and r regions, then prove that n – e + r = 2. (3)

   b) Apply Dijkstra's Algorithm to find the shortest path from the vertex v_1 to v_4 in the following graph. (5)



   c) What do you mean by isolated vertex and pendant vertex ? (2)

Answer **any one** question out of question no. 3 - 4. (1 × 15)

3. a) For the four sets A, B, C and D. Confirm or disprove the following identities :
   $(A-B)\times(C-D)=(A\times C)-(B\times D)$. (3)

   b) Determine the number of integers between 1 and 200, that are not divisible by any of the integers 2, 3 and 5. (4)

   c) Let the function $f:R\to R$ be defined by

   $$f(x)=\begin{cases}3x-2 & \text{for } x>3\\ 2x^2+3 & \text{for } -2<x\le 3\\ 3x^2-7 & \text{for } x\le -2\end{cases}.$$

   Find $f^{-1}(5)$. (3)

   d) Define poset. (2)

   e) Draw the Hasse-diagram for the poset $(P(S),\subseteq)$, P(S) is the power set on S = {a, b, c}. (3)

4. a) How many solutions are there of x + y + z = 17, subject to the constraint x ≥1, y≥2 and z ≥ 3. (3)

   b) Give the statement of Poisson distribution. (2)

   c) In a test, an examiner either guesses or copies or knows the answer to multiple choice question with four choices, only one answer being correct. The probability that he makes a guess is $\frac{1}{3}$ and probability that he copies the answer is $\frac{1}{6}$. The probability that his answer is correct, given that he copies it, is $\frac{1}{8}$. Find the probability that he knew the answer to the question, given that he correctly answers it. (4)

d) Find a recurrence relation and give initial conditions for the number of bit strings (made by binary digits) of length n that do not contain the pattern 11.
Hence , find the explicit solution of that recurrence relation. (3+3)

Group – B

5. Answer **any two** questions from the following : $(2\times 2\frac{1}{2})$

a) Consider be whose productions are
$S \rightarrow aAS / a,$
$A \rightarrow SbA / SS / ba$
Show that $S \rightarrow aabbaa$ by constructing a derivation tree by rightmost derivation which yields aabbaa.

b) Represent the set of all strings over {x, y} which ends with xx & begins with y.

c) Prove that (1 + 00*1) + (1 + 00*1) (0 + 10*1)* (0 + 10*1) = 0*1(0+10*1)*.

d) Construct a non-deterministic finite automata, which accepts the set of all strings over {0, 1} ending with 010.

Answer **any two** questions from the following : (2 × 10)

6. a) Distinguish between context free & context sensitive grammar. (3)

b) Construct a deterministic finite automata accepting all strings over {a, b} ending with aba or aaba. (7)

7. a) Define non-deterministic finite automata. (3)

b) Construct a context-free grammar to generate $\{a^m b^n \mid 1 \le m \le n\}$. (4)

c) Design a Turing machine that accepts the strings over {a, b} containing even numbers of a's. (4)

8. a) Define pushdown automata. (2)

b) What is the relationship between the linear bounded automata and context-sensitive languages? (2)

c) Write a step-by-step procedure to minimize a given finite automata. (6)

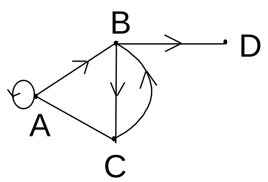
Group – C

9. Answer **any two** questions from the following : $(2\times 2\frac{1}{2})$

a) Define Big-theta.

b) What are the advantages & disadvantages of Strassen's Matrix Multiplication method?

c) Construct the adjacency matrix of the following graph :



Answer **any two** questions from the following : (2 × 10)

10. a) Derive an expression to represent time complexity of the following algorithm :

```
For ( i = 1 to n)
    For ( j = 1 to n)
        C[i, j] = 0
            For k = 1 to n
            C[i, j] = C[i, j] + A[i, k] * B[k, j]
        end for
    end for
end for.
```
(5)

b) Prove or disprove transitive and reflexive property of Big-theta. (5)

11. a) If $f_1(x) = O(g_1(x))$ and $f_2(x) = O(g_2(x))$, prove $(f_1 + f_2)(x)$ is $O(\max(g_1(x), g_2(x)))$. (3)

b) Using Master's method , give tight asymptotic bound for the following recurrence : $T(n) = 4T\left(\frac{n}{2}\right) + n^2$. (3)

c) Stating appropriate logic, formulate recurrence relation of merge sort. (4)

12. a) Derive complexity of merge sort. (3)

b) What are the features of dynamic programming? (2)

c) Write an algorithm for finding the Minimum spanning tree of a graph. Derive its time complexity. (5)

13. a) Write an algorithm for BFS over a graph and illustrate with a suitable example. (6)

b) What is the incidence matrix representation of graph? (2)

c) What is the relationship between P & NP problems? (2)